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Characterization of $\mathbb{B}^{\sigma(\cdot)}_{p,q}(\mathbb{T}^d)$ by a constructive approximation methods

We deal with summation methods for multiple Fourier series in the framework of function spaces with generalized smoothness. The fact that f belongs to the Besov space $\mathbb{B}^{\sigma(\cdot)}_{p,q}(\mathbb{T}^d)$ with generalized smoothness means that

$$\int_0^1 \sigma(t)^q \omega_k(f,\, t)_p^q \frac{dt}{t} \;<\; \infty\,. \tag{$*$}$$

Here $\sigma\,:\,(0,1)\to(0,\infty)$ is an admissible function, $\omega_k(f,\,t)_p$ denotes the modulus of smoothness of order k, where $k>\overline{s}(\sigma)$ ($\overline{s}(\sigma)$ is an upper Boyd index) is a natural number.

The aim of the talk is to give a characterization of $\mathbb{B}^{\sigma(\cdot)}_{p,q}(\mathbb{T}^d)$ by a constructive approximation method which makes sense in $L_p(\mathbb{T}^d)$ for all $p,\,0<p\le\infty$, and all admissible functions. As approximants we consider families of linear polynomial operators. We define

$$\mathcal{L}^{\varphi}_n f(x,\lambda) \;=\; (2n+1)^{-d}\sum_k f(t^k_n+\lambda)\, W^{\varphi}_n(x-t^k_n-\lambda)$$

for $f\in L_p(\mathbb{T}^d),\ (x,\lambda)\in\mathbb{T}^d\times\mathbb{T}^d$, and $n\in\mathbb{N}$. Here, the sum is taken over all $k\in\{0,1,\ldots,2n\}^d$ and $t^k_n=\frac{2\pi}{2n+1}(k_1,\ldots,k_d)$. We are able to prove the equivalence of $(*)$ and

$$\sum_{n=1}^{\infty}\frac{\sigma(n^{-1})^q}{n}\,\|f-\mathcal{L}^{\varphi}_n f\|^q_{\overline{p}} \;<\infty$$

for all admissible functions $\sigma(\cdot)$ with $\overline{s}(\sigma)<\alpha$ (for some α) and $p\in(0,\infty]$. Here $\|\cdot\|_{\overline{p}}$ stands for the (quasi-)norm in $L_p(\mathbb{T}^d\times\mathbb{T}^d)$. The method of proof is based on the characterizations of $\mathbb{B}^{\sigma(\cdot)}_{p,q}(\mathbb{T}^d)$ as approximation spaces as well as by means of generalized K-functionals.

This talk is based on joint works with K. Runovski (Sevastopol) and H.-J. Schmeisser (Jena).