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Boundary value problems with rough boundary data

We consider linear parameter-elliptic boundary value problems of the form

$$\begin{aligned}(\lambda - A)u &= f \quad \text{in } G,\\ B_j u &= g_j \quad (j = 1, \ldots, m) \;\text{ on } \partial G,\end{aligned}$$

in a sufficiently smooth domain $G \subset \mathbb{R}^n$, where A is a partial differential operator of order $2m$ and B_j are boundary operators of order $m_j < 2m$. We assume $f \in L^p(G)$ but the boundary data g_j do not belong to the classical trace space. More precisely, we assume $g_j \in B^s_{pp}(\partial G)$ with $s \leq 2m - m_j - 1/p$, which implies that the classical theory cannot be applied. This question is motivated by problems with boundary noise and/or dynamics on the boundary.
To obtain unique solvability, we consider a class of Sobolev spaces with anisotropic structure which allows us to obtain a generalized boundary trace as well as unique solvability in the half-space and, to some extent, in domains. As an application, we consider boundary value problems with dynamic boundary conditions and show that the related operator generates an analytic semigroup in the product space $L^p(G) \times L^p(\partial G)$. This is the case, e.g., for the Bi-Laplacian with Wentzell boundary conditions or for the linearized Cahn-Hilliard equation with dynamic boundary conditions.

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