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On the randomized complexity of parametric integration

Within the framework of Information-Based Complexity theory (IBC) we study linear problems in the randomized setting using standard information. In other words, we consider randomized versions of sampling numbers. We present examples where adaptive and non-adaptive randomized n-th minimal errors (n-th sampling numbers) deviate by a power of n. This answers an old question of IBC. The result is in contrast to the deterministic setting, where it is well-known that adaptive and non-adaptive n-th minimal errors can deviate at most by a factor of 2.

In fact, the examples are natural problems: parametric integration in Sobolev spaces. More precisely, we determine adaptive and non-adaptive randomized n-th minimal errors of

$$S : W_p^r(D) \to L_q(D_1), \quad (Sf)(s) = \int_{D_2} f(s,t)dt \quad (s \in D_1),$$

where

$$D = [0,1]^d = D_1 \times D_2, \quad D_1 = [0,1]^{d_1}, \quad D_2 = [0,1]^{d_2},$$
$$1 \le p, q \le \infty, \quad d, d_1, d_2, r \in \mathbf{N}, \quad d = d_1 + d_2, \quad \frac{r}{d_1} > \left(\frac{1}{p} - \frac{1}{q}\right)_+ .$$