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Strongly nonlinear multiplicative inequalities with elliptic operators and regularity theory

We are interested in the following inequality, obtained in 2012 together with Jan Peszek:

$$\int_{(a,b)} |f'(x)|^q h(f(x)) dx \leq C \int_{(a,b)} \left(\sqrt{|f''(x) \mathcal{T}_h(f(x))|} \right)^q h(f(x)) dx,$$

as well as its Orlicz variants, where $\mathcal{T}_h(f)$ is certain transformation of function f with the property $\mathcal{T}_{\lambda^\alpha}(f) \sim f$, generalizing previous results in this direction originated by Mazja.

We will discuss further developments of this inequality, focusing on its multidimensional variant obtained recently with Tomas Roskovec and Dalimil Pesa:

$$\int_{\Omega} |\nabla f(x)|^q h(f(x)) dx \leq C \int_{\Omega} \left(\sqrt{|Pf(x) \mathcal{T}_h(f(x))|} \right)^q h(f(x)) dx,$$

which involves the elliptic operator P . We will present some applications of such inequalities to the regularity theory for the nonlinear PDE's of elliptic type and focus on certain open problems in the theory of function spaces, related to the undertaken issue.

The talk will be based on the the chain of my joint works obtained together with Katarzyna Pietruska-Pałuba, Jan Peszek, Katarzyna Mazowiecka, Tomasz Choczewski, Alberto Fiorenza and Claudia Capogne, Tomas Roskovec and Dalimil Pesa.