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On the regularity of Brownian motion

We are interested on the regularity of the paths of the Brownian motion $(W_t)_{t\in I}$, where $I = [0,1]$. We denote for a positive function g on I by $\mathcal{C}^g([0,1])$ the Hölder space as the collection of all functions $f : I \to \mathbb{R}$ which satisfy

$$|f(x) - f(y)| \leq cg(|x-y|) \qquad \text{for all } 0 \leq x, y \leq 1.$$

Already in 1937 Lévy showed that the paths of (W_t) lie almost surely in $\mathcal{C}^g([0,1])$ with $g(r) = |r \log r|^{1/2}$. It is known that this is the smallest space in the Hölder scale where the Brownian motion belongs to. The $\log$ factor is necessary since the paths do not belong to the Hölder space with $g(r) = |r|^{1/2}$, i.e. $\mathcal{C}^{1/2}([0,1]) = B^{1/2}_{\infty,\infty}([0,1])$. Ciesielski proved in 1991 that the $\log$ factor in smoothness can be avoided by paying a price in integrability in the scale of Besov spaces when he showed that the paths belong almost surely to $B^{1/2}_{p,\infty}([0,1])$ where $1 \leq p < \infty$.

In the talk I use Lévy's decomposition of the Brownian motion into the Faber system and different characterizations of Besov type spaces into this system to easily show these results. Furthermore, we also generalize this approach to provide new regularity results of the Brownian motion.

These approaches can easily be adapted to show regularity results for higher dimensional Brownian motion, the so called Wiener sheet. Here we need to study function spaces of dominating mixed smoothness.

This talk is based on a joint work in progress with Cornelia Schneider (FAU Erlangen) and Jan Vybíral (TU Prague).