Jonas Sauer

Friedrich Schiller University Jena, Germany

Spaces of Modelled Distributions

In the mathematical analysis of partial differential equations, irregularities of functions and generalized functions can be measured by means of different topologies which all have their justifications in applications. This idea has been rendered more precisely over the past century or so, culminating in a broad theory of function spaces with examples being Hölder spaces, Besov spaces or Triebel-Lizorkin spaces. The norms of these function spaces can typically be regarded as some sort of comparison of the local behaviour of a given function to polynomials. Virtually all modern theory of deterministic quasilinear parabolic PDEs exploits the full force of this theory of function spaces by combining results on restrictions and extensions, embeddings, trace operators, weights and interpolation properties.

On the other hand, Hairer's celebrated theory of regularity structures treats singular (stochastic) PDEs and proposes to monitor the irregularity of generalized functions by comparing them to a model that is better adapted to the driving noise than are the mere polynomials. This has led to the notion of spaces of modelled distributions, which are enhancements of their classical counterparts. Such enhanced function spaces provide a finer notion of regularity and make it possible to treat classically ill-defined problems, but a comprehensive theory of such spaces is largely missing in the literature.

In my talk I will present how spaces of modelled distributions naturally occur in the study of singular (stochastic) PDEs and present first interpolation results.

Parts of the talk are based on joint works with Felix Otto, Scott Smith and Hendrik Weber.