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## On Haar frames in Sobolev type spaces

We study the behavior of the Haar system in Besov and Triebel-Lizorkin spaces on the real line for a parameter range in which unconditionality does not hold. First, we obtain a range of parameters, extending up to smoothness s < 1, in which the spaces  $F_{p,q}^s$  and  $B_{p,q}^s$  are characterized in terms of doubly oversampled Haar coefficients. Secondly, in the case that 1/p < s < 1 and  $f \in B_{p,q}^s$ , we actually prove that the usual Haar coefficient norm,  $\|\{2^j\langle f, h_{j,\mu}\rangle\}_{j,\mu}\|_{b_{p,q}^s}$  remains equivalent to  $\|f\|_{B_{p,q}^s}$ . At the endpoint case s = 1 and  $q = \infty$ , we show that such an expression gives an equivalent norm for the Sobolev space  $W_p^1$ , 1 , which is related to a classical result by Bočkarev. Finally, in various endpoint cases we clarify the relation between dyadic and standard Besov and Triebel-Lizorkin spaces.

This is joint work with G. Garrigos and A. Seeger.