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Characterization of Sobolev functions with zero traces via the distance function from the boundary

Consider a regular domain $\Omega \subset \mathbb{R}^N$ and let $d(x) = \operatorname{dist}(x, \partial\Omega)$. Denote $L_a^{1,\infty}(\Omega)$ the space of functions from $L^{1,\infty}(\Omega)$ having absolutely continuous quasinorms. This set is essentially smaller than $L^{1,\infty}(\Omega)$ but, at the same time, essentially larger than any $L^{1,q}(\Omega)$, $q \in [1, \infty)$.
A classical result of late 1980's states that, for $p \in (1, \infty)$ and $m \in \mathbb{N}$, u belongs to the Sobolev space $W_0^{m,p}(\Omega)$ if and only if $u/d^m \in L^p(\Omega)$ and $|\nabla^m u| \in L^p(\Omega)$. During the consequent decades, several authors have spent considerable effort in order to relax the characterizing condition. Recently, it was proved that $u \in W_0^{m,p}(\Omega)$ if and only if $u/d^m \in L^1(\Omega)$ and $|\nabla^m u| \in L^p(\Omega)$. We will present a new, yet more relaxed condition on the function u/d, namely $u/d \in L_a^{1,\infty}(\Omega)$, which together with $|\nabla u| \in L^p(\Omega)$ still guarantees that $u \in W_0^{1,p}(\Omega)$. Moreover, we will point out a counterexample which demonstrates that after relaxing the condition $u/d \in L_a^{1,\infty}(\Omega)$ to $u/d \in L^{1,\infty}(\Omega)$, the equivalence no longer holds, and we will discuss several regularity conditions for domains.