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Generalized Besov-Triebel-Lizorkin type spaces

Let $0 < p < \infty$, $0 < q \leq \infty$ and $s \in \mathbb{R}$. We introduce a new type of generalized Besov-Triebel-Lizorkin type spaces $A^{s,\varphi}_{p,q}(\mathbb{R}^n)$, where φ belongs to the class $\mathcal{G}_p$, that is, $\varphi : (0,\infty) \to [0,\infty)$ is increasing and $t^{-n/p}\varphi(t)$ is decreasing in $t > 0$.

We start from the well-known Besov-Triebel-Lizorkin type spaces $A^{s,\tau}_{p,q}(\mathbb{R}^n)$, $\tau \geq 0$, and replace $|Q|^\tau$ in their definition by $\varphi(\ell(Q))$, where Q is some dyadic cube with volume $|Q|$ and side length $\ell(Q)$. We establish several basic properties of the spaces $A^{s,\varphi}_{p,q}(\mathbb{R}^n)$ and investigate the relations within that scale of spaces, as well as to some classical function spaces, especially (generalized) Besov-Triebel-Lizorkin-Morrey spaces. Our intention is to study embeddings between spaces of type $A^{s,\varphi}_{p,q}$, and then to apply our findings to study embeddings of generalized Triebel-Lizorkin-Morrey spaces.