

22

FSA-2020

International Conference on Function Spaces and Applications

Oct 01- Oct 07, 2022 | Apolda, Germany

CONFERENCE ABSTRACTS





DFG

International Conference on **Function Spaces and Applications**

Apolda/Thür. (Germany)

October 1 - 7, 2022

Welcome

Dear Colleague,

it is our great pleasure to welcome you at the 'Hotel am Schloss' in Apolda to take part in the FSA-2020 conference. We are all aware that already the title is misleading, as the conference takes place in 2022. Due to the pandemic we had to postpone it by two years – though our hope and confidence, to organise a 'traditional' conference on site was obviously disappointed due to several reasons. We had more than 100 registrations for the first attempt in 2020, but now we are very lucky that more than 60 participants can take part in Apolda, some more – mainly our colleagues from China and Japan – will take part online, at least to some extent.

The aim of the conference is to concentrate on new developments and results in the theory of function spaces and its applications. Just to recall: the conference has two preceding events. First, it will continue the conference 'New perspectives in the theory of function spaces and their applications' (NPFSA-2017) held in September 2017 in Będlewo (Poland). Secondly, it is part of a joint Chinese-German research project in the theory of function spaces. In that sense our conference succeeds the 'International Conference on Function Spaces and Geometric Analysis and Their Applications' held at the Nankai University, Tianjin (China) in October 2019.

The topics we have in mind are:

- new smoothness and regularity concepts,
- high-dimensional approximation and integration,
- approximation of solutions to operator equations,
- existence and regularity of solutions of (non)-linear PDE,
- compactness and concentration phenomena in function spaces,
- function spaces in complexity.

We are glad that we succeeded to bring together many young and established scientists from various fields and from many countries to present their latest results, to exchange new ideas and to step forward collaboration. So it is essentially you that will make the conference a success in the end!

Nevertheless we would like to take this opportunity to thank our main sponsors and some colleagues. The conference is sponsored by the **German Research Foundation (DFG)**, and the **Friedrich Schiller University Jena**. Our project partners from the **Beijing Normal University** supported us in the organisation. It is a pity that they cannot attend the conference in person.

We are especially indebted to **Julia Flatjord** who created all the nice pictures and posters for us.

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Scientific Committee

- Marcin Bownik (Oregon)
- Dorothee D. Haroske (Jena)
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- Hans-Jürgen Schmeißer (Jena)
- Manuela Scheffel (Jena)
- Winfried Sickel (Jena)
- Kristóf Szarvas (Jena)

Lecture halls

- A main lecture hall ('Decke Pitter'), all plenary and online talks, first floor
- B seminar room ('Apollo I'), ground floor
- C seminar room ('Apollo III'), ground floor

The conference office ('Michel') is also located in the ground floor.

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For more information please visit: <http://fsa2020.uni-jena.de/>

Editorial Deadline: **September 20, 2022**

Sunday, October 2, 2022

9.00– 9.15	Opening	
9.15– 10.15	Fernando Cobos (Universidad Complutense de Madrid) <i>'Interpolation of compact bilinear operators by the real and the complex method'</i>	▷ 8
10.15– 11.15	Eiichi Nakai (Ibaraki University) <i>'Multiple Fourier series of some radial functions and lattice point problems'</i>	▷ 9
Coffee break		
11.45– 12.30	Dachun Yang (Beijing Normal University) <i>'John–Nirenberg–Campanato Spaces'</i>	▷ 29
Lunch break		
14.30– 15.15	Maria Ragusa (University of Catania) <i>'Regularity results for minimizers of some variational integrals'</i>	▷ 23
15.15– 16.00	Marcin Bownik (University of Oregon, Eugene) <i>'Parseval wavelet frames on Riemannian manifold'</i>	▷ 14
Coffee break		

Parallel Afternoon Sessions, Lecture halls **A**, **B** and **C**

16.30– 17.00	A	Javier Soria (Universidad Complutense de Madrid) <i>'Discrete Hardy-type inequalities and optimal constants'</i>	▷ 25
	B	Franz Gmeineder (University of Konstanz) <i>'Traces for L^1-based function spaces'</i>	▷ 16
	C	Nick Lindemulder (Radboud University Nijmegen) <i>'The trace method for ℓ^q-structured interpolation'</i>	▷ 21
17.00– 17.30	A	Amiran Gogatishvili (Czech Academy of Sciences Prague) <i>'Almost-compact and compact embeddings of variable exponent spaces'</i>	▷ 16
	B	Dariusz Bugajewski (Adam Mickiewicz University Poznań) <i>'Characterization of compactness in the space of functions of bounded variation'</i>	▷ 14
	C	Sebastian Bechtel (TU Delft) <i>'The Poincaré inequality in uniform domains'</i>	▷ 13
17.30– 18.00	A	Gowri Sankara Raju Kosuru (Indian Institute of Technology Ropar) <i>'Decreasing Rearrangements and Lorentz variant Herz Space'</i>	▷ 20
	B	Oscar Guzman (ECCI University) <i>'Sobolev meets Riesz: an alternative characterization of weighted Sobolev spaces via weighted Riesz bounded variation spaces'</i>	▷ 17
	C	Markus Hansen (Philipps University Marburg) <i>'Wavelets on the sphere: A group-theoretic approach'</i>	▷ 17

Monday, October 3, 2022

9.00 – 10.00	María Carro (University Complutense of Madrid)	▷ 8
	<i>'Solving the Dirichlet and the Neumann problem at the end-point'</i>	
10.00 – 11.00	Luboš Pick (Charles University Prague)	▷ 9
	<i>'Fractional Orlicz-Sobolev spaces'</i>	
<i>Coffee break</i>		
11.30 – 12.15	Qingying Xue (Beijing Normal University)	▷ 28
	<i>'The composition of rough singular integral operators on function spaces'</i>	
<i>Lunch break</i>		
14.30 – 15.30	Sergey Tikhonov (ICREA, Centre de Recerca Matemàtica)	▷ 10
	<i>'Truncated smooth function spaces'</i>	
15.30 – 16.15	Thomas Hempfling (Springer Nature Switzerland)	▷ 18
	<i>'News and developments at Birkhäuser and Springer'</i>	
<i>Coffee break</i>		

Parallel Afternoon Sessions, Lecture halls **A**, **B** and **C**

16.30 – 17.00	A	Robert Denk (University of Konstanz)	▷ 16
		<i>'Boundary value problems with rough boundary data'</i>	
	B	Susana D. Moura (University of Coimbra)	▷ 22
		<i>'Embeddings between generalised Besov-Morrey spaces'</i>	
	C	Alexander Meskhi (Kutaisi International University)	▷ 22
		<i>'Operators of Harmonic Analysis in Grand Variable Exponent Morrey Spaces'</i>	
17.00 – 17.30	A	Herbert Koch (University of Bonn)	▷ 20
		<i>'Conserved energies for the Gross-Pitaevskii equation'</i>	
	B	Helena Gonçalves (University of Aveiro)	▷ 17
		<i>'Embeddings of Besov-type and Triebel-Lizorkin-type spaces on domains'</i>	
	C	Alexandre Almeida (University of Aveiro)	▷ 12
		<i>'The maximal operator in variable exponent Stummel spaces'</i>	
17.30 – 18.00	A	Jonas Sauer (Friedrich Schiller University Jena)	▷ 24
		<i>'Spaces of Modelled Distributions'</i>	
	B	Zhen Liu (Friedrich Schiller University Jena)	▷ 21
		<i>'Generalized Besov-Triebel-Lizorkin type spaces'</i>	
	C	Sorina Barza (Karlstad University)	▷ 13
		<i>'Optimal domain for discrete Hardy minus identity operator'</i>	

Tuesday, October 4, 2022

9.00– 9.45	Yoshihiro Sawano (Chuo University) <i>'Harmonic analysis and maximal regularity'</i>	▷ 24
10.00– 11.00	Leszek Skrzypczak (Adam Mickiewicz University Poznań) <i>'Nuclearity of some classical operators in function spaces'</i>	▷ 10
<i>Coffee break</i>		
11.30– 12.15	Yurii Kolomoitsev (University of Göttingen) <i>'Sharp L_p-error estimates for approximation by sampling operators'</i>	▷ 20
<i>Lunch break</i>		
13.30– 18.30	Excursion	

Wednesday, October 5, 2022

9.00– 10.00	Mark Veraar (Technical University Delft) <i>'The role of vector-valued function spaces in evolution equations'</i>	▷ 11
10.00– 11.00	Mario Ullrich (Johannes Kepler University Linz) <i>'On sampling numbers in L_2'</i>	▷ 11
<i>Coffee break</i>		
11.30– 12.15	Wen Yuan (Beijing Normal University) <i>'Brezis-Van Schaffingen-Yung Formulae in the Setting of Ball Banach Function Spaces'</i>	▷ 29
<i>Lunch break</i>		
14.30– 15.15	Przemysław Wojtaszczyk (Polish Academy of Sciences) <i>'Stable nonlinear manifold and Lipschitz widths'</i>	▷ 28
15.15– 16.00	Stefan Heinrich (University of Kaiserslautern) <i>'On the randomized complexity of parametric integration'</i>	▷ 18
<i>Coffee break</i>		

Parallel Afternoon Sessions, Lecture halls **A**, **B** and **C**

16.30– 17.00	A Thomas Kühn (University of Leipzig) <i>'High-dimensional approximation in periodic function spaces'</i>	▷ 21
	B Anca-Nicoleta Marcoci (Techn. Univ. of Civil Engineering Bucharest) <i>'Schur multipliers techniques in some problems of solid spaces'</i>	▷ 22
	C Cornelia Schneider (Friedrich Alexander Univ. Erlangen-Nuremberg) <i>'Regularity in Besov spaces of parabolic PDEs'</i>	▷ 24

17.00 – 17.30	A	Markus Passenbrunner (Johannes Kepler University Linz) <i>'An algebraic characterization of B-splines'</i>	▷ 23
	B	Liviu-Gabriel Marcoci (Techn. Univ. of Civil Engineering Bucharest) <i>'On some inequalities in stratified Lie groups'</i>	▷ 22
	C	Flóra O. Szemenyei (Friedrich Alexander Univ. Erlangen-Nuremberg) <i>'Besov regularity of elliptic and parabolic PDEs with inhomogeneous boundary conditions on Lipschitz domains'</i>	▷ 26
17.30 – 18.00	A	Winfried Sickel (Friedrich Schiller University Jena) <i>'Pointwise Multipliers for Besov Spaces with $0 < p \leq 1$ - a Wavelet Approach'</i>	▷ 25
	B	Artur Stabuszewski (Warsaw University of Technology) <i>'Embeddings of Slobodeckij spaces on metric-measure spaces'</i>	▷ 25
	C	Henning Kempka (EAH – University of Applied Sciences Jena) <i>'On the regularity of Brownian motion'</i>	▷ 19

Thursday, October 6, 2022

9.00 – 10.00		Agnieszka Kałamańska (University of Warsaw) <i>'Strongly nonlinear multiplicative inequalities with elliptic operators and regularity theory'</i>	▷ 9
10.00 – 11.00		Eero Saksman (University of Helsinki) <i>'Regularity of Hardy Chaos'</i>	▷ 10
<i>Coffee break</i>			
11.30 – 12.15		Jiman Zhao (Beijing Normal University) <i>'Multilinear spectral multipliers on Lie groups'</i>	▷ 29
<i>Lunch break</i>			
14.30 – 15.15		António Caetano (University of Aveiro) <i>'A fractal approach to acoustic scattering by fractal screens'</i>	▷ 15
15.15 – 16.00		Petru A. Cioica-Licht (University of Kassel) <i>'Function spaces for the analysis of stochastic partial differential equations on non-smooth domains'</i>	▷ 15
<i>Coffee break</i>			

Parallel Afternoon Sessions, Lecture halls **A**, **B** and **C**

16.30 – 17.00	A	Markus Weimar (University of Würzburg) <i>'Optimal approximation of break-of-scale embeddings'</i>	▷ 27
	B	Hana Turčinová (Charles University Prague) <i>'Characterization of Sobolev functions with zero traces via the distance function from the boundary'</i>	▷ 26
	C	Pankaj Jain (South Asian University, New Delhi) <i>'On Hausdorff Operators in the framework of weighted Lebesgue and grand Lebesgue spaces'</i>	▷ 19

17.00– 17.30	A	Marc Hovemann (Philipps University Marburg) <i>'Quarklet Characterizations for Triebel-Lizorkin spaces'</i>	▷ 18
	B	Dalimil Peša (Charles University Prague) <i>'Wiener-Luxemburg amalgam spaces'</i>	▷ 23
	C	Kristóf Szarvas (Friedrich Schiller University Jena) <i>'New atomic and molecular decomposition for variable Triebel-Lizorkin spaces and their growth envelope function'</i>	▷ 26

Plenary Session

17.30– 18.15	Tino Ullrich (Technical University Chemnitz) <i>'On Haar frames in Sobolev type spaces'</i>	▷ 27
18.15	Closing	

not yet scheduled:

	Paul Abaivoh (University of Ibadan) <i>'Applications of L^p-Spaces as Partial*-Algebras'</i>	▷ 12
	Sergei Artamonov (Moscow) <i>'Characterization of $\mathbb{B}_{p,q}^{\sigma(\cdot)}(\mathbb{T}^d)$ by a constructive approximation method'</i>	▷ 13

Invited Lectures

María Carro

University Complutense of Madrid, Spain

Monday, 9.00–10.00, A

Solving the Dirichlet and the Neumann problem at the end-point

In 1980 C. Kenig proved that for every Lipschitz domain Ω in the plane there exists $1 \leq p_0 < 2$ so that the Dirichlet problem has a solution for every $f \in L^p(ds)$ and every $p \in (p_0, \infty)$. Moreover, if $p_0 > 1$, the result is false for $p \leq p_0$. The goal of this talk is to analyze what happens at the endpoint L^{p_0} ; that is, we want to look for spaces $X \subset L^{p_0}$ so that the Dirichlet problem has a solution for every $f \in X$. These spaces X will be either a Lorentz space $L^{p_0,1}(ds)$ or some Orlicz space of logarithmic type.

Similar results will be presented for the Neumann problem.

This is a joint work with Virginia Naibo and Carmen Ortiz-Caraballo.

Fernando Cobos

Universidad Complutense de Madrid, Spain

Sunday, 9.15–10.15, A

Interpolation of compact bilinear operators by the real and the complex method

In the last few years it has been shown that compact bilinear operators occur rather naturally in harmonic analysis (see for example [1]). This has motivated the investigation on the behaviour under interpolation of compact bilinear operators. In this talk I will review some recent results of the joint papers with Fernández-Cabrera and Martínez [2, 3, 4] on the behaviour of compact bilinear operators under the real and the complex method.

References.

- [1] Á. Bényi and R.H. Torres, Compact bilinear operators and commutators, Proc. Amer. Math. Soc. 141 (2013) 3609-3621.
- [2] F. Cobos, L.M. Fernández-Cabrera and A. Martínez, Interpolation of compact bilinear operators among quasi-Banach spaces and applications, Math. Nachr. 291 (2018) 2168-2187.
- [2] F. Cobos, L.M. Fernández-Cabrera and A. Martínez, On compactness results of Lions-Peetre type for bilinear operators, Nonlinear Anal. 199 (2020) 111951.
- [4] F. Cobos, L.M. Fernández-Cabrera and A. Martínez, Compactness interpolation results for bilinear operators of convolution type and for operators of product type, J. Approx. Theory 274 (2022) 105688.

Strongly nonlinear multiplicative inequalities with elliptic operators and regularity theory

We are interested in the following inequality, obtained in 2012 together with Jan Peszek:

$$\int_{(a,b)} |f'(x)|^q h(f(x)) dx \leq C \int_{(a,b)} \left(\sqrt{|f''(x) \mathcal{T}_h(f(x))|} \right)^q h(f(x)) dx,$$

as well as in its Orlicz variants, where $\mathcal{T}_h(f)$ is a certain transformation of function f with the property $\mathcal{T}_{\lambda^\alpha}(f) \sim f$, generalizing previous results in this direction originated by Mazja.

We will discuss further developments of this inequality, focusing on its multidimensional variant obtained recently with Tomáš Roskovec and Dalimil Peša:

$$\int_{\Omega} |\nabla f(x)|^q h(f(x)) dx \leq C \int_{\Omega} \left(\sqrt{|Pf(x) \mathcal{T}_h(f(x))|} \right)^q h(f(x)) dx,$$

which involves the elliptic operator P . We will present some applications of such inequalities to the regularity theory for the nonlinear PDE's of elliptic type and focus on certain open problems in the theory of function spaces, related to the undertaken issue.

The talk will be based on the the chain of my joint works obtained together with Katarzyna Pietruska-Pałuba, Jan Peszek, Katarzyna Mazowiecka, Tomasz Choczewski, Alberto Fiorenza and Claudia Capogno, Tomáš Roskovec and Dalimil Peša.

Multiple Fourier series of some radial functions and lattice point problems

For the multiple Fourier series of radial functions, we investigate the behavior of the spherical partial sum. We show convergence properties and singular phenomena for the multiple Fourier series. As singular phenomena, we show the Gibbs-Wilbraham phenomenon, the Pinsky phenomenon and the third phenomenon. The third phenomenon is closely related to the lattice point problems, which is a classical theme of the analytic number theory. Our results on the convergence of the Fourier series are obtained by using the best estimates up to now on lattice point problems. Therefore, if the lattice point problems will be improved in the future, then our results can be also improved.

This is a joint work with Professor Shigehiko Kuratubo (Hirosaki University).

Fractional Orlicz-Sobolev spaces

We shall give a survey of various results on fractional Orlicz-Sobolev spaces obtained recently jointly with Angela Alberico (CNR Naples), Andrea Cianchi (University of Florence), and Lenka Slavíková (Charles University).

Eero Saksman

University of Helsinki, Finland

Thursday, 10.00–11.00, A

On regularity of solutions of Beltrami equations up to the boundary

For certain ranges of parameters we provide regularity results for solutions of planar Beltrami equations. The results are optimal also in terms of the regularity of the boundary.

Based on joint work with Kari Astala (U. Helsinki) and Marti Prats (U. Barcelona).

Leszek Skrzypczak

Adam Mickiewicz University Poznań, Poland

Tuesday, 10.00–11.00, A

Nuclearity of some classical operators in function spaces

Nuclear operators were introduced by A. Grothendieck in 1955. They are a subclass of a family of linear compact operators and their eigenvalues are square summable. During this talk we consider two classes of classical operators acting in function spaces: embeddings of Sobolev type and Fourier transforms. We find sufficient and necessary conditions for the operators to be nuclear when they act between Besov or Triebel-Lizorkin spaces. We consider different situations when the Sobolev embeddings are compact: spaces on (quasi-)bounded domains, weighted spaces and spaces with a radial symmetry condition. Our method of prove is based on the wavelet characterization of the function spaces. As a by-product we prove a vector-valued version of Tong's theorem about nuclearity of embeddings of ℓ_p sequence spaces.

All the presented results are joint works with Dorothee D. Haroske, Hans-Gerd Leopold and Hans Triebel.

References.

- [1] D.D. Haroske, H.-G. Leopold, and L. Skrzypczak. Nuclear embeddings in general vector-valued sequence spaces with an application to Sobolev embeddings of function spaces on quasi-bounded domains. *J. Complexity*, 69:101605, 2022.
- [2] D.D. Haroske, L. Skrzypczak. Nuclear embeddings in weighted function spaces. *Integr. Equ. Oper. Theory* **92** (2020), Article no: 46, 37 pages.
- [3] D. D. Haroske, L. Skrzypczak and H. Triebel Nuclear Fourier transforms, arXiv:2205.03128v1

Sergey Tikhonov

ICREA, Centre de Recerca Matemàtica, Spain

Monday, 14.30–15.30, A (online)

Truncated smooth function spaces

We introduce a new scale of smooth function spaces – truncated spaces. For truncated Besov and Triebel-Lizorkin spaces we discuss their main properties: embeddings, interpolation, duality, lifting, traces.

This is joint work with Óscar Domínguez.

On sampling numbers in L_2

Let $F \subset L_2$ be a class of complex-valued functions on a set D , such that, for all $x \in D$, point evaluation $f \mapsto f(x)$ is a continuous linear functional. We study the L_2 -approximation of functions from F and want to compare the power of function values with the power of arbitrary linear information. To be precise, the *sampling number* $g_n(F)$ is the minimal worst-case error (in F) that can be achieved with n function values, whereas the *approximation number* (or Kolmogorov width) $d_n(F)$ is the minimal worst-case error that can be achieved with n pieces of arbitrary linear information (like derivative values or Fourier coefficients).

Here, we report on recent developments in this problem and, in particular, explain how the individual contributions from [1-4] lead to the following statement:

There is a universal constant $c \in \mathbb{N}$ such that the sampling numbers of the unit ball F of every separable reproducing kernel Hilbert space are bounded by

$$g_{cn}(F) \leq \sqrt{\frac{1}{n} \sum_{k \geq n} d_k(F)^2}.$$

We also obtain similar upper bounds for more general classes F , and provide examples where our bounds are attained up to a constant. For example, if we assume that $d_n(F) \asymp n^{-\alpha}(\log n)^\beta$ for some $\alpha > 1/2$ and $\beta \in \mathbb{R}$, then we obtain

$$g_n(F) \asymp d_n(F),$$

showing that function values are (up to constants) as powerful as arbitrary linear information. The results rely on the solution to the Kadison-Singer problem, which we extend to the subsampling of a sum of infinite rank-one matrices.

References.

- [1] M. Dolbeault, D. Krieg and M. Ullrich, A sharp upper bound for sampling numbers in L_2 , preprint.
- [2] D. Krieg and M. Ullrich, Function values are enough for L_2 -approximation, *Found. Comput. Math.* **21** (2021), 1141–1151.
- [3] D. Krieg and M. Ullrich, Function values are enough for L_2 -approximation: Part II, *J. Complexity* **66** (2021).
- [4] N. Nagel, M. Schäfer and T. Ullrich, A new upper bound for sampling numbers, *Found. Comput. Math.* **21** (2021).

The role of vector-valued function spaces in evolution equations

For many years vector-valued function spaces have played an important role in the analysis of partial differential equations. In the talk I will give an overview of this topic, and present several recent developments. In particular, I will discuss new results on trace theory and interpolation theory of vector-valued function spaces, and present new regularity properties for solutions to PDEs.

Talks

Paul Abaivoh

University of Ibadan, Nigeria

not yet scheduled

Applications of L^p -Spaces as Partial*-Algebras

This article is a study of L^p -spaces defined and characterized as partial*-algebras. Precisely, this class of function spaces is studied here as Quasi*-algebra with a specific focus on CQ^* -algebras, the commutative and non-commutative CQ^* -algebras. Very important properties of these Quasi*-algebras are outlined and characterized, in some cases, as results. In conclusion, applications have been made, anchored on the rich properties, both in quantum field theory and statistical mechanics.

Alexandre Almeida

University of Aveiro, Portugal

Monday, 17.00–17.30, C

The maximal operator in variable exponent Stummel spaces

In this talk we consider variable exponent Stummel spaces and discuss the boundedness of the Hardy-Littewood maximal operator in such spaces. Our results include also the behaviour of the maximal operator in some vanishing Stummel subspaces.

The talk is based on joint work with Humberto Rafeiro.

Characterization of $\mathbb{B}_{p,q}^{\sigma(\cdot)}(\mathbb{T}^d)$ by a constructive approximation method

We deal with summation methods for multiple Fourier series in the framework of function spaces with generalized smoothness. The fact that f belongs to the Besov space $\mathbb{B}_{p,q}^{\sigma(\cdot)}(\mathbb{T}^d)$ with generalized smoothness means that

$$\int_0^1 \sigma(t)^q \omega_k(f, t)_p^q \frac{dt}{t} < \infty. \quad (*)$$

Here $\sigma : (0, 1] \rightarrow (0, \infty)$ is an admissible function, $\omega_k(f, t)_p$ denotes the modulus of smoothness of order k , where $k > \bar{s}(\sigma)$ ($\bar{s}(\sigma)$ is an upper Boyd index) is a natural number.

The aim of the talk is to give a characterization of $\mathbb{B}_{p,q}^{\sigma(\cdot)}(\mathbb{T}^d)$ by a constructive approximation method which makes sense in $L_p(\mathbb{T}^d)$ for all p , $0 < p \leq \infty$, and all admissible functions. As approximants we consider families of linear polynomial operators. We define

$$\mathcal{L}_n^\varphi f(x, \lambda) = (2n + 1)^{-d} \sum_k f(t_n^k + \lambda) W_n^\varphi(x - t_n^k - \lambda)$$

for $f \in L_p(\mathbb{T}^d)$, $(x, \lambda) \in \mathbb{T}^d \times \mathbb{T}^d$, and $n \in \mathbb{N}$. Here, the sum is taken over all $k \in \{0, 1, \dots, 2n\}^d$ and $t_n^k = \frac{2\pi}{2n+1}(k_1, \dots, k_d)$. We are able to prove the equivalence of (*) and

$$\sum_{n=1}^{\infty} \frac{\sigma(n^{-1})^q}{n} \|f - \mathcal{L}_n^\varphi f\|_p^q < \infty$$

for all admissible functions $\sigma(\cdot)$ with $\bar{s}(\sigma) < \alpha$ (for some α depending on φ) and $p \in (0, \infty]$. Here $\|\cdot\|_p$ stands for the (quasi-)norm in $L_p(\mathbb{T}^d \times \mathbb{T}^d)$. The method of proof is based on the characterizations of $\mathbb{B}_{p,q}^{\sigma(\cdot)}(\mathbb{T}^d)$ as approximation spaces as well as by means of generalized K -functionals.

This talk is based on joint works with K. Runovski (Sevastopol) and H.-J. Schmeisser (Jena).

Optimal domain for discrete Hardy minus identity operator

We characterize the optimal domains for the discrete Hardy minus identity operator as well as for discrete dual Hardy minus identity operator, in the weighted Lebesgue spaces of sequences. We also address this question for some weak-type spaces.

The presentation is based on a research paper written together with Javier Soria (University Complutense of Madrid.)

The Poincaré inequality in uniform domains

We consider a uniform domain in Euclidean space. The central question in this talk is to establish a local Poincaré inequality, that is to say, to estimate the p -average of the oscillation of a function on some ball by the p -average of its gradient on a ball with the same center but with a radius that is a (fixed) multiple of the original radius. Our motivation for this investigation are p -bounds for Riesz transforms associated with a non-smooth elliptic operator in divergence form. Our approach is based on a Sobolev extension operator that has homogeneous and local estimates. The construction is derived from similar ideas by P. W. Jones.

Parseval wavelet frames on Riemannian manifold

We construct Parseval wavelet frames in $L^2(M)$ for a general Riemannian manifold M and we show the existence of wavelet unconditional frames in $L^p(M)$ for $1 < p < \infty$. This is made possible thanks to smooth orthogonal projection decomposition of the identity operator on $L^2(M)$. We also show a characterization of Triebel-Lizorkin $F_{p,q}^s(M)$ and Besov $B_{p,q}^s(M)$ spaces on compact manifolds in terms of magnitudes of coefficients of Parseval wavelet frames. We achieve this by showing that Hestenes operators are bounded on $F_{p,q}^s(M)$ and $B_{p,q}^s(M)$ spaces on manifolds M with bounded geometry.

This talk is based on a joint work with Karol Dzielniak and Anna Kamont.

Characterization of compactness in the space of functions of bounded variation

As far as we know the only one characterization of compactness in the space of functions of bounded variation in the sense of Jordan is given in Dunford and Schwartz's celebrated monograph [2] (Exercise IV.13.48) – however, in our opinion, it is not too useful from the point of view of applications.

In this talk we are going to present a new compactness criterion for subsets of the Banach space of functions of bounded variation in the sense of Jordan (briefly: $BV(I)$, where I is a compact interval of \mathbb{R}), which seems to be more comfortable from that point of view. It is based on the notion of an equivariated subset of that space. We also present how one can use that notion to define a quasimeasure of noncompactness in the space $BV(I)$.

Finally, we will focus on new general compactness criteria in normed spaces from the paper [4], which are based the notion of an equinormed set using a suitable family of semi-norms which satisfy some natural conditions.

The results presented in this talk come from the papers [3] and [4]. One can also see the paper [5] for new such type results in Lipschitz spaces. Moreover, we refer the reader interested in applications of BV type spaces to the recently published monograph [1].

References.

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- [5] J. Gulgowski, P. Kasprzak, P. Maćkowiak, *Compactness in Lipschitz spaces and around*, submitted.

A fractal approach to acoustic scattering by fractal screens

Sound-soft fractal screens can scatter acoustic waves even when they have zero surface measure. We report how to solve such scattering problems by a novel application of the boundary element method (BEM) where each BEM basis function is supported in a fractal set, and the integration involved in the formation of the BEM matrix is with respect to a non-integer order Hausdorff measure. In this way, for a class of fractals that are attractors of iterated function systems, convergence rates for the BEM can be proved under certain natural regularity assumptions on the solution of the underlying boundary integral equation.

This is joint work with S. N. Chandler-Wilde (Reading), A. Gibbs (UCL), D. P. Hewett (UCL) and A. Moiola (Pavia).

Function spaces for the analysis of stochastic partial differential equations on non-smooth domains

In this talk I will present a class of weighted Sobolev spaces that have been recently used to analyse the regularity of stochastic partial differential equations (SPDEs) on dihedral angles and polygons. The weights are based on both the distance to the boundary and the distance to the vertex(es) of the domain. The spaces are designed in such a way that they accurately capture both effects: the singularities of the solution caused by the irregularity of the boundary and the singularities caused by the noise. I will focus on basic properties of the spaces and on their suitability for the analysis of SPDEs. Moreover, I will discuss possible extensions of the theory to three-dimensional polyhedral cones.

This is joint work with Markus Weimar (RUB Bochum, DE) and Cornelia Schneider (FAU Nuremberg, DE) as well as Kyeong-Hun Kim (Korea U, KR), Kijung Lee (Ajou U, KR), and Felix Lindner (U Kassel, DE).

Boundary value problems with rough boundary data

We consider linear parameter-elliptic boundary value problems of the form

$$\begin{aligned}(\lambda - A)u &= f \quad \text{in } G, \\ B_j u &= g_j \quad (j = 1, \dots, m) \quad \text{on } \partial G,\end{aligned}$$

in a sufficiently smooth domain $G \subset \mathbb{R}^n$, where A is a partial differential operator of order $2m$ and B_j are boundary operators of order $m_j < 2m$. We assume $f \in L^p(G)$ but the boundary data g_j do not belong to the classical trace space. More precisely, we assume $g_j \in B_{pp}^s(\partial G)$ with $s \leq 2m - m_j - 1/p$, which implies that the classical theory cannot be applied. This question is motivated by problems with boundary noise and/or dynamics on the boundary.

To obtain unique solvability, we consider a class of Sobolev spaces with anisotropic structure which allows us to obtain a generalized boundary trace as well as unique solvability in the half-space and, to some extent, in domains. As an application, we consider boundary value problems with dynamic boundary conditions and show that the related operator generates an analytic semi-group in the product space $L^p(G) \times L^p(\partial G)$. This is the case, e.g., for the Bi-Laplacian with Wentzell boundary conditions or for the linearized Cahn-Hilliard equation with dynamic boundary conditions.

This talk is based on joint work with David Ploß, Sophia Rau (both Konstanz), and Jörg Seiler (Torino).

Traces for L^1 -based function spaces

In this talk, we discuss optimal trace estimates in L^1 -based function spaces. Here the differentiability is not determined by full k -th order gradients, but only certain differential expressions belonging to L^1 . Proceeding in this way, we obtain the first generalization of Aronszajn's coercive inequalities for linear systems with L^1 -data on domains, but also a new approach to the classical Uspenskii theorem on sharp Besov traces in the higher order case.

The results presented in this talk are based on joint work with J. Van Schaftingen (U Louvain) and B. Raita (U Pisa).

Almost-compact and compact embeddings of variable exponent spaces

We give a necessary and sufficient condition for the almost compact embeddings of one variable exponent Lebesgue space into another. We apply such results to characterize compact embeddings of variable exponent Sobolev spaces into variable exponent Lebesgue spaces.

Keywords: almost-compact embeddings, Banach function spaces, variable Lebesgue spaces, variable Sobolev spaces

2010 Mathematics Subject Classification: 46E30, 26D15

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Embeddings of Besov-type and Triebel-Lizorkin-type spaces on domains

Besov-type spaces $B_{p,q}^{s,\tau}(\mathbb{R}^d)$ and Triebel-Lizorkin-type spaces $F_{p,q}^{s,\tau}(\mathbb{R}^d)$, $0 < p < \infty$ (or $p = \infty$ in the B-case), $0 < q \leq \infty$, $\tau \geq 0$, $s \in \mathbb{R}$, are part of a class of function spaces built upon Morrey spaces $\mathcal{M}_{u,p}(\mathbb{R}^d)$, $0 < p \leq u < \infty$. For this reason, they are usually called in the literature as *smoothness spaces of Morrey type* or, shortly, *smoothness Morrey spaces*. The increasing study in the last decades is motivated primarily by possible applications. We mention partial differential equations, such as (fractional) Navier-Stokes equations.

In this talk, we present embeddings of Besov-type and Triebel-Lizorkin-type spaces on a bounded domain $\Omega \subset \mathbb{R}^d$,

$$\text{id}_\tau: B_{p_1,q_1}^{s_1,\tau_1}(\Omega) \hookrightarrow B_{p_2,q_2}^{s_2,\tau_2}(\Omega) \quad \text{and} \quad \text{id}_\tau: F_{p_1,q_1}^{s_1,\tau_1}(\Omega) \hookrightarrow F_{p_2,q_2}^{s_2,\tau_2}(\Omega).$$

Aiming at the continuity and compactness of id_τ , we give necessary and sufficient conditions for such properties to be achieved.

This talk is based on a joint work with D. D. Haroske and L. Skrzypczak.

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Sobolev meets Riesz: an alternative characterization of weighted Sobolev spaces via weighted Riesz bounded variation spaces

We introduce weighted Riesz bounded variation spaces defined in an open subset of the n -dimensional Euclidean space. Using the newly introduced space we give a characterization of weighted Sobolev spaces when the weight belongs to the Muckenhoupt class. Some embeddings and weak type estimates are also provided. We also provide, as an application of the main result, a characterization of variable exponent Sobolev spaces via variable exponent Riesz bounded variation spaces.

This is a joint work with Professor David Cruz-Urbe and Professor Humberto Rafeiro.

Wavelets on the sphere: A group-theoretic approach

Based on ideas by Antoine and Vandergheynst, continuous wavelet frames on the sphere are constructed from a single so-called admissible function by applying the unitary operators associated to a representation of the Lorentz group, which is square-integrable modulo the nilpotent factor of the Iwasawa decomposition. We then prove necessary and sufficient conditions for functions to be admissible, strengthening corresponding results by Antoine and Vandergheynst. Additionally, we show how the resulting conditions can be satisfied with the help of the Stereographic projection.

On the randomized complexity of parametric integration

Within the framework of Information-Based Complexity theory (IBC) we study linear problems in the randomized setting using standard information. In other words, we consider randomized versions of sampling numbers. We present examples where adaptive and non-adaptive randomized n -th minimal errors (n -th sampling numbers) deviate by a power of n . This answers an old question of IBC. The result is in contrast to the deterministic setting, where it is well-known that adaptive and non-adaptive n -th minimal errors can deviate at most by a factor of 2.

In fact, the examples are natural problems: parametric integration in Sobolev spaces. More precisely, we determine adaptive and non-adaptive randomized n -th minimal errors of

$$S : W_p^r(D) \rightarrow L_q(D_1), \quad (Sf)(s) = \int_{D_2} f(s, t) dt \quad (s \in D_1),$$

where

$$D = [0, 1]^d = D_1 \times D_2, \quad D_1 = [0, 1]^{d_1}, \quad D_2 = [0, 1]^{d_2}, \\ 1 \leq p, q \leq \infty, \quad d, d_1, d_2, r \in \mathbf{N}, \quad d = d_1 + d_2, \quad \frac{r}{d_1} > \left(\frac{1}{p} - \frac{1}{q}\right)_+.$$

News and developments at Birkhäuser and Springer

We present selected publications and give some insights into recent developments in publishing, particularly those using artificial intelligence / machine learning.

Quarklet Characterizations for Triebel-Lizorkin spaces

In this talk we present a characterization in terms of quarklets for the Triebel-Lizorkin spaces. For that purpose in a first step we define the Triebel-Lizorkin spaces $F_{r,q}^s(\mathbb{R})$. Here we use the Fourier analytical approach and work with a smooth dyadic decomposition of the unity. In a second step we construct the quarklets. To this end we deal with cardinal B-splines. They can be used to assemble biorthogonal compactly supported B-spline wavelets. Here we apply the construction of Cohen, Daubechies and Feauveau presented in 1992. Those wavelets can be employed to obtain the quarklets. For that purpose we enrich the B-spline wavelets with polynomials of degree $p \in \mathbb{N}_0$. In the third part of the talk we use the quarklets to characterize the Triebel-Lizorkin spaces. So it turns out that under some conditions on the parameters a function belongs to a Triebel-Lizorkin space $F_{r,q}^s(\mathbb{R})$ if and only if it can be represented in terms of quarklets. In connection with that we also obtain a new equivalent quasinorm for the Triebel-Lizorkin spaces.

On Hausdorff Operators in the framework of weighted Lebesgue and grand Lebesgue spaces

In this talk, we shall discuss the weighted L^p -boundedness of the Hausdorff operator

$$(H_\phi f)(x) := \int_0^\infty \frac{\phi(y)}{y} f\left(\frac{x}{y}\right) dy.$$

As an application of Sawyer's duality principle, the corresponding boundedness for monotone functions will be derived. Also, we shall discuss the operator H_ϕ in the framework of grand Lebesgue spaces. We shall point out the possibility of dealing with the more general Dunkl-Hausdorff operator

$$(H_{\alpha,\phi} f)(x) := \int_0^\infty \frac{\phi(y)}{y^{2\alpha+2}} f\left(\frac{x}{y}\right) dy.$$

On the regularity of Brownian motion

We are interested in the regularity of the paths of the Brownian motion $(W_t)_{t \in I}$, where $I = [0, 1]$. We denote for a positive function g on I by $\mathcal{C}^g([0, 1])$ the Hölder space as the collection of all functions $f : I \rightarrow \mathbb{R}$ which satisfy

$$|f(x) - f(y)| \leq cg(|x - y|) \quad \text{for all } 0 \leq x, y \leq 1.$$

Already in 1937 Lévy showed that the paths of (W_t) lie almost surely in $\mathcal{C}^g([0, 1])$ with $g(r) = |r \log r|^{1/2}$. It is known that this is the smallest space in the Hölder scale where the Brownian motion belongs to. The log factor is necessary since the paths do not belong to the Hölder space with $g(r) = |r|^{1/2}$, i.e. $\mathcal{C}^{1/2}([0, 1]) = B_{\infty, \infty}^{1/2}([0, 1])$. Ciesielski proved in 1991 that the log factor in smoothness can be avoided by paying a price in integrability in the scale of Besov spaces when he showed that the paths belong almost surely to $B_{p, \infty}^{1/2}([0, 1])$ where $1 \leq p < \infty$.

In the talk I use Lévy's decomposition of the Brownian motion into the Faber system and different characterizations of Besov type spaces into this system to easily show these results. Furthermore, we also generalize this approach to provide new regularity results of the Brownian motion.

These approaches can easily be adapted to show regularity results for higher dimensional Brownian motion, the so called Wiener sheet. Here we need to study function spaces of dominating mixed smoothness.

This talk is based on a joint work in progress with Cornelia Schneider (FAU Erlangen) and Jan Vybřal (TU Prague).

Herbert Koch

University of Bonn, Germany

Monday, 17.00–17.30, A

Conserved energies for the Gross-Pitaevskii equation

The Gross-Pitaevskii equation is essentially the defocusing Nonlinear Schrödinger equation with nontrivial conditions at infinity. It admits dark soliton solutions. I will explain the geometric and topological structure of the finite energy space for the Gross-Pitaevskii equation. The GP equation has a Lax pair. An approximate diagonalization of the Lax equation is a basis for the construction of conserved energies E^s for all $s \geq 0$.

This is joint work with Xian Liao.

Yurii Kolomoitsev

University of Göttingen, Germany

Tuesday, 11.30–12.15, A

Sharp L_p -error estimates for approximation by sampling operators

We study approximation properties of linear sampling operators in the spaces L_p . By means of the Steklov averages, we introduce a new measure of smoothness containing simultaneously information on the smoothness of a function in L_p and discrete information on this function at sampling points. The new measure of smoothness enables us to improve and extend several classical results of approximation theory to the case of linear sampling operators. In particular, we obtain matching direct and inverse approximation inequalities for sampling operators in L_p , find the exact order of decreasing of the corresponding L_p -errors for particular classes of functions, and introduce a special K -functional and its realization suitable for studying smoothness properties of sampling operators. In this talk, we will present some of these results in the case of approximation of functions by trigonometric Lagrange interpolation polynomials.

This is a joint work Tetiana Lomako (University of Göttingen).

Gowri Sankara Raju Kosuru

Indian Institute of Technology Ropar, India

Sunday, 17.30–18.00, A

Decreasing Rearrangements and Lorentz variant Herz Space

The notion of decreasing rearrangements is an important tool in analysis, playing a vital role in various inequalities. This notion allows us to construct a non-negative, non-decreasing function f^* on the interval $[0, \infty)$ for every measurable function f on any arbitrary σ -finite measure space. The decreasing function f^* encodes information about the properties of the original function f . One remarkable application of this notion is the discovery of Lorentz spaces. On the other hand the Herz space $K_{p,q}^a$ was introduced in connection with the Lipschitz spaces and last few decades, these spaces have been extensively studied in various directions. In this talk we refine the classical Herz spaces $K_{p,q}^a$ by introducing a class of functions called Herz-Lorentz spaces $HL_{p,q}^{a,r}$. The authors study a few properties these spaces in the framework of Banach function spaces. We also discuss the completeness of these spaces and give some embedding results.

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Thomas Kühn
University of Leipzig, Germany

Wednesday, 16.30–17.00, A

High-dimensional approximation in periodic function spaces

First I will introduce Gevrey spaces $G^{s,c}(\mathbb{T}^d)$, a new family of function spaces on the d -dimensional torus. They are intimately related to the famous Gevrey classes, which were defined by Maurice Gevrey in 1918, who was motivated by applications to PDEs.

Next I will discuss approximation numbers a_n of embeddings of the Gevrey spaces $G^{s,c}(\mathbb{T}^d)$ resp. Sobolev spaces of dominating mixed smoothness $H_{\text{mix}}^r(\mathbb{T}^d)$ into $L_2(\mathbb{T}^d)$. The main focus will be on sharp asymptotic rates, existence of asymptotic constants, d -dependence (as $d \rightarrow \infty$), and also on the behaviour in the preasymptotic range $n \leq 2^d$. Clearly, for computational aspects of high-dimensional approximation problems, it is essential to have good bounds for small n , rather than to know 'only' the exact asymptotic as $n \rightarrow \infty$.

Finally, if time allows, I will give an interpretation of these results in terms of tractability notions from Information-Based Complexity, where some interesting phenomena occur.

The talk is based on recent joint work with Martin Petersen (Leipzig), Winfried Sickel (Jena) and Tino Ullrich (Chemnitz).

Nick Lindemulder
Radboud University Nijmegen, Netherlands

Sunday, 16.30–17.00, C

The trace method for ℓ^q -structured interpolation

In this talk we discuss the trace method within the framework of the sequentially structured interpolation method for sequence structures that are of ℓ^q -type. Our setting covers the ℓ^q -interpolation method introduced by Kunstmann, which for the scale of Triebel-Lizorkin spaces plays the role the real interpolation method plays for the scale of Besov spaces. As an application we consider the problem of maximal L^p - L^q -regularity for parabolic boundary value problems.

The talk is based on joint work with Emiel Lorist (Delft University of Technology).

Zhen Liu
Friedrich Schiller University Jena, Germany

Monday, 17.30–18.00, B

Generalized Besov-Triebel-Lizorkin type spaces

Let $0 < p < \infty$, $0 < q \leq \infty$ and $s \in \mathbb{R}$. We introduce a new type of generalized Besov-Triebel-Lizorkin type spaces $A_{p,q}^{s,\varphi}(\mathbb{R}^n)$, where φ belongs to the class \mathcal{G}_p , that is, $\varphi : (0, \infty) \rightarrow [0, \infty)$ is increasing and $t^{-n/p}\varphi(t)$ is decreasing in $t > 0$.

We start from the well-known Besov-Triebel-Lizorkin type spaces $A_{p,q}^{s,\tau}(\mathbb{R}^n)$, $\tau \geq 0$, and replace $|Q|^\tau$ in their definition by $\varphi(\ell(Q))$, where Q is some dyadic cube with volume $|Q|$ and side length $\ell(Q)$. We establish several basic properties of the spaces $A_{p,q}^{s,\varphi}(\mathbb{R}^n)$ and investigate the relations within that scale of spaces, as well as to some classical function spaces, especially (generalized) Besov-Triebel-Lizorkin-Morrey spaces. Our intention is to study embeddings between spaces of type $A_{p,q}^{s,\varphi}$, and then to apply our findings to study embeddings of generalized Triebel-Lizorkin-Morrey spaces.

Anca-Nicoleta Marcoci

Wednesday, 16.30–17.00, B

Technical University of Civil Engineering Bucharest, Romania

Schur multipliers techniques in some problems of solid spaces

In this talk, we present recent results concerning the solid hull and the largest solid subspace of some spaces of infinite matrices. In particular, we complete a result due to A. Pelczynski and F. Sukochev.

Liviu-Gabriel Marcoci

Wednesday, 17.00–17.30, B

Technical University of Civil Engineering Bucharest, Romania

On some inequalities in stratified Lie groups

The first proof in the Euclidean setting of so called improved Sobolev inequalities is due to P. Gerard, F. Oru and Y. Meyer. On the other hand, standard Sobolev inequalities have been studied by A. Cianchi in the context of Orlicz-Sobolev spaces. In this talk we present some improved Sobolev inequalities in the framework of stratified Lie groups.

Alexander Meskhi

Monday, 16.30–17.00, C

Kutaisi International University, Georgia

Operators of Harmonic Analysis in Grand Variable Exponent Morrey Spaces

The boundedness of the Hardy–Littlewood maximal, Calderón–Zygmund singular and fractional integral operators in grand variable exponent Morrey spaces under log-Hölder continuity condition on exponents is established. Grand variable exponent Morrey space is a non-standard function space unifying variable and grand Morrey spaces. We study the problem for operators and spaces defined on quasi-metric measure spaces with doubling measure (space of homogeneous type). Sobolev-type inequality for fractional integrals with variable parameters in these spaces defined on quasi-metric measure spaces with non-doubling measure (non-homogeneous space) is also derived. The results are new for maximal, Calderón–Zygmund singular and fractional integral operators, and grand variable exponent Morrey spaces defined, for example, on certain domains in Euclidean spaces, bounded rectifiable curves satisfying the regularity condition, nilpotent Lie groups with Haar measure (homogeneous groups), etc. Some structural properties of grand variable exponent Morrey spaces are also investigated.

Susana D. Moura

Monday, 16.30–17.00, B

University of Coimbra, Portugal

Embeddings between generalised Besov-Morrey spaces

We present sufficient and necessary conditions for the embeddings between generalised Besov-Morrey spaces. Embeddings of the Besov-Morrey spaces into the Lebesgue spaces are also considered. Our approach requires a wavelet characterisation of the spaces which we establish for the system of Daubechies wavelets.

This is joint work with Dorothee Haroske (Jena) and Leszek Skrzypczak (Poznan).

An algebraic characterization of B-splines

B-splines of order k can be viewed as a mapping N taking a $(k+1)$ -tuple of increasing real numbers $a_0 < \dots < a_k$ and giving as a result a certain piecewise polynomial function. Looking at this mapping N as a whole, basic properties of B-spline functions imply that it has the following algebraic properties: (1) $N(a_0, \dots, a_k)$ has local support contained in the interval $[a_0, a_k]$; (2) $N(a_0, \dots, a_k)$ allows refinement, i.e. for every $a \in \cup_{j=0}^{k-1} (a_j, a_{j+1})$ we have that if $(\alpha_0, \dots, \alpha_{k+1})$ is the increasing rearrangement of the points $\{a_0, \dots, a_k, a\}$, the 'old' function $N(a_0, \dots, a_k)$ is a linear combination of the 'new' functions $N(\alpha_0, \dots, \alpha_k)$ and $N(\alpha_1, \dots, \alpha_{k+1})$; (3) N is translation and dilation invariant. It is easy to see that derivatives of $N(a_0, \dots, a_k)$ satisfy properties (1)–(3) as well.

Let F be a mapping taking $(k+1)$ -tuples of increasing real numbers to some generalized function. In this paper we show that under some additional mild condition on the size of the supports of $F(a_0, \dots, a_k)$ relative to the interval $[a_0, a_k]$, properties (1)–(3) are already sufficient to deduce that $F(a_0, \dots, a_k)$ is a non-zero multiple of (some derivative of) a B-spline function. However, and somewhat surprisingly, we explicitly give examples of choices of F satisfying (1)–(3) but are not of this form.

This is a joint work with Anna Kamont.

Wiener–Luxemburg amalgam spaces

In this talk we introduce the concept of Wiener–Luxemburg amalgam spaces which are a modification of the more classical Wiener amalgam spaces intended to address some of the shortcomings the latter face in the context of rearrangement-invariant Banach function spaces. We present results concerning many of their properties, including (but not limited to) their normability, embeddings between them and their associate spaces.

Regularity results for minimizers of some variational integrals

In an open set $\Omega \subset \mathbb{R}^m$ ($m \geq 2$) let us define the maps $u : \Omega \rightarrow \mathbb{R}^n$. Also, let us consider the $p(x)$ -energy functional as follows

$$\mathcal{E}(u; \Omega) := \int_{\Omega} \left(g^{\alpha\beta}(x) G_{ij}(u) D_{\alpha} u^i(x) D_{\beta} u^j(x) \right)^{p(x)/2} dx,$$

being $(g^{\alpha\beta}(x))$ and $(G_{ij}(u))$ symmetric positive definite matrices whose entries are continuous functions defined on Ω and \mathbb{R}^n respectively, and $p(x)$ a continuous function on Ω with $p(x) \geq 2$.

Main focus is the study of regularity properties, interior and up to the boundary, of the minimizers u of \mathcal{E} and developments in this direction. Some open problems concerning qualitative properties are discussed.

Jonas Sauer

Friedrich Schiller University Jena, Germany

Monday, 17.30–18.00, A

Spaces of Modelled Distributions

In the mathematical analysis of partial differential equations, irregularities of functions and generalized functions can be measured by means of different topologies which all have their justifications in applications. This idea has been rendered more precisely over the past century or so, culminating in a broad theory of function spaces with examples being Hölder spaces, Besov spaces or Triebel-Lizorkin spaces. The norms of these function spaces can typically be regarded as some sort of comparison of the local behaviour of a given function to polynomials. Virtually all modern theory of deterministic quasilinear parabolic PDEs exploits the full force of this theory of function spaces by combining results on restrictions and extensions, embeddings, trace operators, weights and interpolation properties.

On the other hand, Hairer's celebrated theory of regularity structures treats singular (stochastic) PDEs and proposes to monitor the irregularity of generalized functions by comparing them to a model that is better adapted to the driving noise than are the mere polynomials. This has led to the notion of spaces of modelled distributions, which are enhancements of their classical counterparts. Such enhanced function spaces provide a finer notion of regularity and make it possible to treat classically ill-defined problems, but a comprehensive theory of such spaces is largely missing in the literature.

In my talk I will present how spaces of modelled distributions naturally occur in the study of singular (stochastic) PDEs and present first interpolation results.

Parts of the talk are based on joint works with Felix Otto, Scott Smith and Hendrik Weber.

Yoshihiro Sawano

Chuo University, Japan

Tuesday, 9.00–9.45, A (online)

Harmonic analysis and maximal regularity

The aim of this talk is to discuss the maximal regularity of the heat equation in Morrey spaces based on the 2010 paper by Ogawa and Shimizu. What differs from their work is that the Fourier multipliers are used instead of interpolation. Some recent studies on the real interpolation show that Morrey spaces do not interpolate well. The estimate needed in this paper is the local means. The local means is transformed into the form which is suitable in the context of the maximal regularity. As an application, the Cauchy problems for the Keller-Segel system are studied.

The function spaces used in this paper correspond to the scaling critical case for the Keller-Segel system. Some observation shows that why Besov-Morrey spaces are suitable for the study of maximal regularity and that some other related spaces are not.

Cornelia Schneider

Friedrich Alexander University of Erlangen-Nuremberg, Germany

Wednesday, 16.30–17.00, C

Regularity in Besov spaces of parabolic PDEs

This talk is concerned with the regularity of solutions to parabolic evolution equations. Special attention is paid to the smoothness in a specific scale of Besov spaces, the so-called adaptivity scale. The regularity in these spaces determines the approximation order that can be achieved by adaptive approximation schemes. As a special example we highlight the heat equation.

Winfried Sickel

Friedrich Schiller University Jena, Germany

Wednesday, 17.30–18.00, A

Pointwise Multipliers for Besov Spaces with $0 < p \leq 1$ - a Wavelet Approach

In 1992, in his famous book on wavelets, Y. Meyer gave a characterization of the set of all pointwise multipliers $M(B_{1,1}^0(\mathbb{R}^d))$ of the Besov space $B_{1,1}^0(\mathbb{R}^d)$ in terms of wavelet coefficients. We will discuss an extension of the Meyer characterization to all Besov spaces $B_{p,p}^s(\mathbb{R}^d)$, $s \in \mathbb{R}$, $0 < p \leq 1$. For $s > d(\frac{1}{p} - 1)$ several different characterizations of $M(B_{p,p}^s(\mathbb{R}^d))$ have been found by Maz'ya, Shaposhnikova ($p = 1$), Netrusov, Triebel and Nguyen, Sickel. We plan to make a short comparison. Finally, we will discuss the Fourier analytic approach. This will allow us to identify $M(B_{p,p}^s(\mathbb{R}^d))$ (in some cases) as an intersection of $L_\infty(\mathbb{R}^d)$ with certain Morrey smoothness spaces.

The talk is based on joint work with Dachun Yang and Wen Yuan (Beijing Normal University).

Artur Stąbuszewski

Warsaw University of Technology, Poland

Wednesday, 17.30–18.00, B

Embeddings of Slobodeckij spaces on metric-measure spaces

We consider a fractional Slobodeckij space defined on a metric-measure space (X, d, μ) . During the talk I will present that under some additional assumption, there is an equivalence between boundedness of the Sobolev embedding operator and the lower regularity of μ . Moreover, I will discuss compactness of L^p embedding operator and relation with Hajtász-Sobolev spaces. The talk is based on a joint work with Przemysław Górką.

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Javier Soria

Universidad Complutense de Madrid, Spain

Sunday, 16.30–17.00, A

Discrete Hardy-type inequalities and optimal constants

Recently, several authors have considered the problem of determining optimal norm inequalities for discrete Hardy-type operators (like Cesàro or Copson). In this talk, we will obtain sharp bounds for the norms of the difference of the Cesàro operator with either the identity or the shift operator, when they are restricted to the cone of decreasing sequences in ℓ^p . Finally, we also address the case of weighted inequalities and find an interesting contrast between the norms of these two difference operators.

This is a joint work with Santiago Boza (UPC).

New atomic and molecular decomposition for the variable Triebel-Lizorkin spaces and their growth envelope function

We study unboundedness properties of functions belonging to Triebel-Lizorkin spaces with variable norm using growth envelopes. A new atomic and molecular decomposition will be given for variable Triebel-Lizorkin spaces. The atoms, which are defined, have the property, that their variable Triebel-Lizorkin norms are uniformly bounded. This property of the atoms is used to compute the growth envelope function of variable Triebel-Lizorkin spaces in the sub-critical case. Our results extend the ones for the corresponding classical spaces in a natural way.

Besov regularity of elliptic and parabolic PDEs with inhomogeneous boundary conditions on Lipschitz domains

This short talk is concerned with the regularity of solutions of elliptic and parabolic PDEs with inhomogeneous boundary conditions on bounded polyhedral cones $K \subset \mathbb{R}^3$ and polyhedral domains $D \subset \mathbb{R}^3$, respectively. Special attention is paid to the regularity in the specific scale $B_{\tau,\tau}^\alpha$, $\frac{1}{\tau} = \frac{\alpha}{3} + \frac{1}{p}$ of Besov spaces. The regularity of the solution in these spaces determines the order of approximation that can be achieved by adaptive numerical schemes. For our purposes we consider also weighted Sobolev spaces with mixed weights. These weights measure the distance to the edges and vertices of K and D and can 'compensate' possible singularities of the solutions of the PDEs, which may occur at the boundary of K and D . In particular, as one of our main tools we provide an embedding between the weighted Sobolev and Besov spaces, which afterwards allows us to investigate the regularity of the solutions in the Besov scale.

Characterization of Sobolev functions with zero traces via the distance function from the boundary

Consider a regular domain $\Omega \subset \mathbb{R}^N$ and let $d(x) = \text{dist}(x, \partial\Omega)$. Denote $L_a^{1,\infty}(\Omega)$ the space of functions from $L^{1,\infty}(\Omega)$ having absolutely continuous quasinorms. This set is essentially smaller than $L^{1,\infty}(\Omega)$ but, at the same time, essentially larger than any $L^{1,q}(\Omega)$, $q \in [1, \infty)$.

A classical result of late 1980's states that, for $p \in (1, \infty)$ and $m \in \mathbb{N}$, u belongs to the Sobolev space $W_0^{m,p}(\Omega)$ if and only if $u/d^m \in L^p(\Omega)$ and $|\nabla^m u| \in L^p(\Omega)$. During the consequent decades, several authors have spent considerable effort in order to relax the characterizing condition. Recently, it was proved that $u \in W_0^{m,p}(\Omega)$ if and only if $u/d^m \in L^1(\Omega)$ and $|\nabla^m u| \in L^p(\Omega)$. We will present a new, yet more relaxed condition on the function u/d , namely $u/d \in L_a^{1,\infty}(\Omega)$, which together with $|\nabla u| \in L^p(\Omega)$ still guarantees that $u \in W_0^{1,p}(\Omega)$. Moreover, we will point out a counterexample which demonstrates that after relaxing the condition $u/d \in L_a^{1,\infty}(\Omega)$ to $u/d \in L^{1,\infty}(\Omega)$, the equivalence no longer holds, and we will discuss several regularity conditions for domains.

On Haar frames in Sobolev type spaces

We study the behavior of the Haar system in Besov and Triebel-Lizorkin spaces on the real line for a parameter range in which unconditionality does not hold. First, we obtain a range of parameters, extending up to smoothness $s < 1$, in which the spaces $F_{p,q}^s$ and $B_{p,q}^s$ are characterized in terms of doubly oversampled Haar coefficients. Secondly, in the case that $1/p < s < 1$ and $f \in B_{p,q}^s$, we actually prove that the usual Haar coefficient norm, $\|\{2^j \langle f, h_{j,\mu} \rangle\}_{j,\mu}\|_{b_{p,q}^s}$ remains equivalent to $\|f\|_{B_{p,q}^s}$. At the endpoint case $s = 1$ and $q = \infty$, we show that such an expression gives an equivalent norm for the Sobolev space W_p^1 , $1 < p < \infty$, which is related to a classical result by Bočkarev. Finally, in various endpoint cases we clarify the relation between dyadic and standard Besov and Triebel-Lizorkin spaces.

This is joint work with G. Garrigos and A. Seeger.

Optimal approximation of break-of-scale embeddings

As a rule of thumb in approximation theory, the asymptotic speed of convergence of numerical methods is governed by the regularity of the objects we like to approximate. Besides classical isotropic Sobolev smoothness, the notion of dominating mixed regularity of functions turned out to be an important concept in numerical analysis. Although approximation rates of embeddings *within* the scales of isotropic or dominating-mixed L_p -Sobolev spaces are well-understood, not that much is known for embeddings *across* those scales. In this talk we introduce particular instances of new hybrid smoothness spaces which cover both scales as special cases. Moreover, we present (non-)adaptive wavelet-based approximation algorithms that achieve optimal dimension-independent rates of convergence for certain practically important break-of-scale embeddings.

The talk is based on recent joint work [1] with Janina Hübner (RUB) and Glenn Byrenheid (FSU Jena).

References.

- [1] G. Byrenheid, J. Hübner, and M. Weimar. Rate-optimal sparse approximation of compact break-of-scale embeddings. Submitted (2022). arXiv:2203.10011

Stable nonlinear manifold and Lipschitz widths

Dozens of widths are known, see e.g. [1]. Generally they are sequences of numbers that measure the best, *possible under given restrictions*, approximation of the set $\mathcal{K} \subset X$ where \mathcal{K} is (usually) a compact subset of a Banach space X .

In recent decades in numerical analysis we see the growing interest in non-linear algorithms. While it is well known that nonlinear methods of approximation can often perform dramatically better than linear methods, there are still questions on how to measure the optimal performance possible for such methods. Some attempts were made in [2] however they were not taking into account the numerical stability of the methods. In the talk I present two types of widths taking this into account.

In [3] we introduce stable manifold width

$$\delta_{n,\gamma}(\mathcal{K}) = \inf_{a,M,\|\cdot\|_Y} \sup_{k \in \mathcal{K}} \|f - M(a(k))\|_X$$

where $a : \mathcal{K} \rightarrow \mathbb{R}^n$, $M : \mathbb{R}^n \rightarrow X$, $\|\cdot\|_Y$ is a norm on \mathbb{R}^n and both a and M are γ -Lipschitz maps when we use $\|\cdot\|_Y$ on \mathbb{R}^n . In [4] we introduce Lipschitz widths

$$d_n^\gamma(\mathcal{K}) = \inf_{\|\cdot\|_Y} \inf_{\Phi_n} \sup_{k \in \mathcal{K}} \inf_{y \in B_n} \|k - \Phi_n(y)\|_X$$

where $\|\cdot\|_Y$ is a norm on \mathbb{R}^n , B_n is a unit ball in \mathbb{R}^n equipped with norm $\|\cdot\|_Y$ and $\Phi_n : B_n \rightarrow X$ is a γ -Lipschitz map.

In the talk I will try to present

1. The justification for such widths.
2. Basic properties of those new widths and their relations with Banach spaces.
3. Relations between new widths and classical ones especially the entropy numbers.
4. Present some results and examples motivated by important various numerical procedures specially by deep learning.

References.

- [1] A. Pinkus, *n*-widths in Approximation Theory, Springer, 2012
- [2] R. DeVore, R. Howard, C. Micchelli, *Optimal nonlinear approximation*, Manuscripta Mathematica **63**(4) (1989), 469-478
- [3] A. Cohen, R. DeVore, G. Petrova, P. W. *Optimal stable nonlinear approximation*, Found. Comput. Math. **22** (2022), no. 3, 607-648
- [4] G. Petrova, P. W. *Lipschitz widths*, Constructive Approximation (to appear)

The composition of rough singular integral operators on function spaces

The composition of singular integral operators arises typically in the algebra of singular integral (Calderón and Zygmund, 1956) and the non-coercive boundary-value problems for elliptic equations. Considerable attention has been paid to the composition of singular integral operators. In this talk, we will focus on the behavior of the bounds of the composition for rough singular integral operators on the weighted space, on rearrangement-invariant Banach function spaces and on quasi-Banach spaces.

Dachun Yang

Beijing Normal University, People's Republic of China

Sunday, 11.45–12.30, A (online)

John–Nirenberg–Campanato Spaces

In this talk, we introduce some recent developments related to John–Nirenberg spaces and their applications on the Euclidean space or its cubes, including (local) John–Nirenberg–Campanato spaces and vanishing John–Nirenberg spaces. Some open questions are also mentioned.

Wen Yuan

Beijing Normal University, People's Republic of China

Wednesday, 11.30–12.15, A (online)

Brezis-Van Schaftingen-Yung Formulae in the Setting of Ball Banach Function Spaces

Let X be a Ball Banach function space. In this talk, we first recall a surprising formula recently obtained by Brezis-Van Schaftingen-Yung on a new characterization of the Sobolev space $W^{1,1}(\mathbb{R}^n)$. Then, under the mild assumption that the Hardy–Littlewood maximal operator is bounded on the associated space of X , and via some new proofs, we establish the Brezis-Van Schaftingen-Yung formula in a more general setting of Ball Banach function space X . This generalization has a wide range of applications and, particularly, enables us to establish new fractional Sobolev and Gagliardo–Nirenberg inequalities in various function spaces, including Morrey spaces, mixed-norm Lebesgue spaces, variable Lebesgue spaces, weighted Lebesgue spaces, Orlicz spaces, and Orlicz-slice (generalized amalgam) spaces.

Jiman Zhao

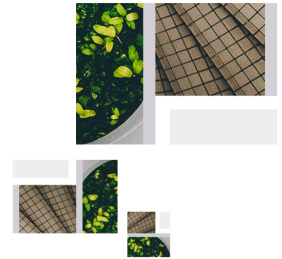
Beijing Normal University, People's Republic of China

Thursday, 11.30–12.15, A (online)

Multilinear spectral multipliers on Lie groups

In this talk, we will introduce some boundedness properties of multilinear spectral multipliers on stratified groups and Lie groups of polynomial growth.

This is joint work with Dr. Jingxuan Fang.



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